Tensor Charges of the Nucleon

in the SU(3) Chiral Quark Soliton Model

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(April 1996)

Abstract

The tensor charges of the nucleon are calculated in the framework of the SU(3) chiral quark soliton model. The rotational $1/N_c$ and strange quark mass corrections are taken into account up to linear order. We obtain the following numerical values of the tensor charges: $\delta u = 1.12$, $\delta d = -0.42$, and $\delta s = -0.008$. In contrast to the axial charges, the tensor charges in our model are closer to those of the nonrelativistic quark model, in particular, the net number of the transversely polarized strange quarks in a transversely polarized nucleon δs is compatible with zero.

PACS:11.15.Pg, 12.40.-y, 13.88.+e, 14.20.Dh

Typeset using REVTEX

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1. There are three different twist-two nucleon parton distributions $f_1(x)$, $g_1(x)$ and $h_1(x)$. The knowledge of them would provide us the complete information about the leading-order hard processes. Two of these distributions– $f_1(x)$ and $g_1(x)$– have been investigated in detail theoretically and measured in deep-inelastic lepton scattering (for a review see [1]). However, the third distribution $h_1(x)$ which is called the transversity distribution is chirally odd, so that it does not appear in inclusive deep-inelastic scattering experiments. It was discussed that $h_1(x)$ can be measured in the Drell-Yan lepton-pair production [2,3], direct photon production, and heavy-quark production in polarized pp collisions [4] and in the pion production in deep inelastic scattering [5]. Recently Bourrely and Soffer suggested that $h_1(x)$ can be determined in the neutral gauge boson Z production in pp collisions [6]. The $h_1(x)$ is totally unknown experimentally, while its measurement has been proposed by the RHIC spin collaboration [7], HERMES collaboration at HERA [8] and more recently by COMPASS collaboration at CERN [9].

Jaffe and Ji [10] demonstrated that the first moment of $h_1(x)$ is related to the tensor charge of the nucleon:

$$\int_0^1 dx \left(h_1(x) - \bar{h}_1(x) \right) = \delta q, \tag{1}$$

where $\bar{h}_1(x)$ is an antiquark transversity distribution, $\bar{h}_1(x) = -h_1(-x)$. The tensor charges δq are defined as the forward nucleon matrix element:

$$\langle N | \bar{\psi}_q \sigma_{\mu\nu} \psi_q | N \rangle = \delta q \bar{U} \sigma_{\mu\nu} U, \tag{2}$$

where q denotes the flavour index ($q = u, d, s$) and $U(p)$ stands for a Dirac spinor and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. We introduce for convenience a flavour–singlet and two octet tensor charges:

$$g_{\rm T}^{(0)} = \delta u + \delta d + \delta s, \tag{3}$$

$$g_{\rm T}^{(3)} = \delta u - \delta d, \tag{4}$$

$$g_{\rm T}^{(8)} = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s). \tag{5}$$

In contrast to the axial charges the tensor ones depend on the renormalization scale already at one–loop level. The corresponding anomalous dimension has been evaluated in

Refs. [12–14]: $\gamma = 2\alpha_s/3\pi$. However, their dependence on the normalization point is very weak:

$$\delta q(\mu^2) \;=\; \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\frac{4}{33-2N_f}} \delta q(\mu_0^2). \tag{6}$$

As $\mu \to \infty$ the $\delta q(\mu^2)$ is slowly vanishing. This equation can be used to evolve the tensor charges from the low normalization point (several hundreds MeV) pertinent to the chiral quark-soliton model (χQSM) we are dealing with, to higher normalization points. Since the corresponding anomalous dimension is relatively small, the value of the tensor charge at a higher normalization point is insensitive to uncertainties of low normalization points relevant to our model [15].

Quite recently, we have examined the tensor charges of the nucleon in the framework of the SU(2) chiral quark-soliton model (χQSM) and suggested the mechanism as to how the tensor charges are different from the axial ones [15]. In the present paper, we extend the former investigation to the case of three flavours. This enables us to evaluate the net number of the transversely polarized up, down and strange quarks in a transversely polarized nucleon separately.

Since the tensor current is not related to any symmetry, it can not be constructed as a Noether current. Hence, it is not obvious how to build up the tensor current in the Skyrme model, because the corresponding Lagrangian consists only of mesonic fields. In contrast to the Skyrme model, one can define unambiguously any quark current in the χQSM having explicit quark degrees of freedom.

2. The χQSM is based on the interaction of dynamically massive constituent quarks with pseudo-Goldstone meson fields. It is characterized by the low-energy effective chiral lagrangian given by the functional integral over quark (ψ) in the background pion field [16–19]:

$$\exp\left(iS_{\text{eff}}[\pi(x)]\right) \;=\; \int \mathcal{D}\psi \mathcal{D}\bar\psi \exp\left(\int d^4x \bar\psi D \psi\right), \tag{7}$$

where D is the Dirac operator

$$D \;=\; i\slashed{\partial} - \hat{m} - MU^{\gamma_5}. \tag{8}$$

3

U^{γ_5} denotes the pseudoscalar chiral field

$$U^{\gamma_5} \; = \; \exp i\pi^a \lambda^a \gamma_5 \; = \; \frac{1+\gamma_5}{2} U \; + \; \frac{1-\gamma_5}{2} U^\dagger. \tag{9}$$

$\hat{m}$ is the matrix of the current quark masses $\hat{m} \; = \; \mathrm{diag}(m_u, m_d, m_s)$ and λ^a represent the usual Gell-Mann matrices. The M stands for the dynamical quark mass arising as a result of the spontaneous chiral symmetry breaking.

The effective chiral action given by Eq. (7) is known to contain automatically the Wess–Zumino term and the four-derivative Gasser–Leutwyler terms, with correct coefficients. Therefore, at least the first four terms of the gradient expansion of the effective chiral lagrangian are correctly reproduced by Eq. (7), and chiral symmetry arguments do not leave much freedom for further modifications. Eq. (7) has been derived from the instanton model of the QCD vacuum [19], which provides a natural mechanism of chiral symmetry breaking and enables one to express the dynamical mass M and the ultraviolet cutoff Λ intrinsic in Eq. (7) through the Λ_{QCD} parameter. It should be mentioned that Eq. (7) is of a general nature: one need not believe in instantons and still use Eq. (7). The effective chiral theory Eq. (7) is valid for the values of the quark momenta up to the ultraviolet cutoff Λ. Therefore, in using Eq. (7) we imply that we are computing the tensor charges at the normalization point about $\Lambda \approx 600$ MeV.

An immediate application of the effective chiral theory Eq. (7) is the quark-soliton model of baryons [20]. According to these ideas the nucleon can be viewed as a bound state of N_c (=3) *valence* quarks kept together by a hedgehog-like pion field whose energy coincides by definition with the aggregate energy of quarks from the negative Dirac sea. Such a semiclassical picture of the nucleon is justified in the limit $N_c \to \infty$ – in line with more general arguments by Witten [24]. Roughly speaking, the χQSM builds a bridge between the naive valence quark model of baryons and the Skyrme model. The further studies showed that the χQSM is successful in reproducing the static properties and form factors of the baryons using just one parameter set and adjusted in the mesonic sector to m_π, f_π and m_K. (see the recent review [25]).

The forward nucleon matrix element Eq. (2) in the rest frame of the nucleon is nonzero only for indices μ, ν being space-like $i, j = 1, 2, 3$. Using a γ-matrix property $\sigma_{\mu\nu} = (i/2)\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5$, we can relate the operator in the left-hand side of Eq. (2) in the rest frame of the nucleon to $\bar{\psi}\sigma_{0i}\gamma_5\lambda^a\psi$. Hence, the tensor charges can be calculated as a nucleon forward matrix element of $\bar{\psi}\gamma_0\gamma_5\gamma_k\lambda^a\psi$. It is interesting to notice that the only difference between the axial and tensor charges is the γ_0 matrix. It implies that in the nonrelativistic quark model (NRQM) the tensor charges coincide with the axial ones [10,11]:

$$
\begin{aligned}
\delta u = \Delta u &= \frac{4}{3}, \\
\delta d = \Delta d &= -\frac{1}{3}, \\
\delta s = \Delta s &= 0.
\end{aligned}
\tag{10}
$$

The tensor charges of the nucleon can be related to the following correlation function in Euclidean space:

$$
- i\langle 0| J_N(\vec{y}, \frac{T}{2})\psi^\dagger\gamma_0\gamma_k\lambda^a\psi J_N^\dagger(\vec{x}, -\frac{T}{2})|0\rangle
\tag{11}
$$

at large Euclidean time T. The nucleon current J_N is built of N_c quark fields:

$$
J_N(x) = \frac{1}{N_c!}\epsilon_{i_1\cdots i_{N_c}}\Gamma^{\alpha_1\cdots\alpha_{N_c}}_{JJ_3TT_3Y}\psi_{\alpha_1 i_1}(x)\cdots\psi_{\alpha_{N_c} i_{N_c}}(x).
\tag{12}
$$

$\alpha_1\cdots\alpha_{N_c}$ denote spin–flavour indices, while $i_1\cdots i_{N_c}$ designate colour indices. The matrices $\Gamma^{\alpha_1\cdots\alpha_{N_c}}_{JJ_3TT_3Y}$ are taken to endow the corresponding current with the quantum numbers JJ_3TT_3Y.

The nucleon matrix element of $\bar{\psi}\gamma_0\gamma_5\gamma_k\lambda^a\psi$ can be computed as the Euclidean functional integral in the χQSM

$$
\begin{aligned}
\langle N|\psi^\dagger\gamma_0\gamma_5\gamma_k\lambda^a\psi|N\rangle = \frac{1}{\mathcal{Z}}\lim_{T\to\infty}& \exp\left(ip_0\frac{T}{2} - ip_0'\frac{T}{2}\right) \\
&\times \int d^3x d^3y \exp\left(-i\vec{p'}\cdot\vec{y} + i\vec{p}\cdot\vec{x}\right)\int \mathcal{D}U\int \mathcal{D}\psi\int \mathcal{D}\psi^\dagger \\
&\times J_N(\vec{y}, T/2)\psi^\dagger\gamma_0\gamma_5\gamma_k\lambda^a\psi J_N^\dagger(\vec{x}, -T/2) \\
&\times \exp\left[\int d^4z\psi^\dagger D\psi\right].
\end{aligned}
\tag{13}
$$

In the large N_c limit the integral over Goldstone fields U can be calculated by the steepest descent method (semiclassical approximation). The corresponding saddle point equation admits a static soliton solution, an example of which is the hedgehog field configuration:

$$U_s(\vec{x}) = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{14}$$

where U_0 is the SU(2) chiral matrix of the form:

$$U_0 = \exp\left[i\vec{n} \cdot \vec{\tau} P(r)\right]. \tag{15}$$

The $P(r)$ denotes the profile function satisfying the boundary condition $P(0) = \pi$ and $P(\infty) = 0$, which is determined by solving the saddle point equations (for details see Ref. [25]). The soliton is quantized by introducing collective coordinates corresponding to $SU(3)_{fl}$ rotations of the soliton in flavour space (and simultaneously $SU(2)_{spin}$ in spin space):

$$U(t, \vec{x}) = R(t)U_s(\vec{x})R^\dagger(t), \tag{16}$$

where $R(t)$ is a time–dependent $SU(3)$ matrix. The quantum states arising from this quantization have the quantum numbers of baryons. In the large N_c limit the soliton angular velocity $\Omega = R^\dagger(t)\dot{R}(t)$ is parametrically small, so that we can use the angular velocity as a small parameter. Recently, it was demonstrated [26,27] that taking into account the first order rotational corrections one can solve old problems of underestimate of the nucleon axial constants and magnetic moments in the chiral soliton model of the nucleon. Also it is worth noting that the correct non-relativistic quark model results for axial and tensor charges Eq. (11) can be obtained in the non-relativistic limit of the χQSM only if the first order rotational corrections are considered [28]. The next source of the corrections to the leading order result is the effects of $SU(3)$ symmetry violation caused by the nonzero strange quark mass. We calculate the $SU(3)$ symmetry breaking corrections linear in m_s.

We follow closely the formalism described in Ref. [25] and hence we present below only the results without any detail (they will be given elsewhere). The tensor charges of the nucleon have the following structure (order of each term in $1/N_c$ and m_s is shown explicitly):

$$g_{\mathrm{T}}^{(a)} = N_c \left\{ \mathcal{T}_0 \langle D_{a3}^{(8)} \rangle_N + \frac{\mathcal{T}_1}{N_c} \langle D_{a3}^{(8)} \rangle_N + \frac{\mathcal{T}_2}{N_c} \langle d_{3pq} D_{ap}^{(8)} J_q \rangle_N + \frac{\mathcal{T}_3}{N_c} \langle D_{a8}^{(8)} J_3 \rangle_N + m_s \mathcal{T}_4 \langle D_{88}^{(8)} D_{a3}^{(8)} \rangle_N \right.$$

$$\left. + \; m_s \mathcal{T}_5 \langle D_{83}^{(8)} D_{a8}^{(8)} \rangle_N + m_s \mathcal{T}_6 \langle d_{3pq} D_{8p}^{(8)} D_{aq}^{(8)} \rangle_N + O(\frac{1}{N_c^2}) + O(\frac{m_s}{N_c}) + O(m_s^2) \right\} \qquad (17)$$

for $a = 3, 8$ and

$$g_{\mathrm{T}}^{(0)} = \sqrt{3} \mathcal{T}_3 \langle J_3 \rangle_N + \sqrt{3} m_s N_c \mathcal{T}_5 \langle D_{83}^{(8)} \rangle_N, \qquad (18)$$

where $\langle O \rangle_N$ denotes the average over rotational state of the quantized soliton corresponding to the nucleon, J_a $(a = 1, \ldots, 8)$ is the operator of infinitesimal $SU(3)$ rotation, for $a = 1, 2, 3$ it coincides with the operator of angular momentum. The quantities $\mathcal{T}_i$ in Eqs.(17, 18) can be calculated as functional traces of the form:

$$\mathcal{T}_i = \mathrm{Sp}(\frac{1}{D(U_s)} \Gamma_1^i \frac{1}{D(U_s)} \Gamma_2^i), \qquad (19)$$

where $D(U_s)$ is a Dirac operator Eq. (8) in the static chiral soliton field Eq. (14), $\Gamma_{1,2}^i$ are operators which are local in coordinate space and generically non-local in time. The explicit expressions for $\mathcal{T}_i$ will appear elsewhere.

In order to evaluate Eqs. (17, 18) numerically, we employ the Kahana-Ripka discretized basis method [22,25]. The constituent quark mass is fixed to 420 MeV in our model by producing best the SU(3) baryon mass splittings [29]. All other relevant static baryon observables and form factors are also well reproduced in the model [25] for this constituent quark mass. To make sure of the numerical calculation, we compare our results for $\mathcal{T}_i$ with the analytical ones of the gradient expansion justified in the limit of large soliton size. Our numerical procedure reproduces within few percent the analytical results of the gradient expansion for each $\mathcal{T}_i$ separately in large soliton size limit.

The results of our calculations are summarized in Tables I–II. We see that the rotational $1/N_c$ corrections are of great importance numerically, whereas the $SU(3)$ symmetry breaking corrections are relatively small. Unlike the axial charges [30,31], the tensor ones in our model are closer to their values in nonrelativistic quark model, in particular the strangeness contribution to the tensor charge δs is compatible with zero, while the analogous contribution

to the axial charge Δs in the same model and in the experiment is negative and distinctive from zero [30].

3. To summarize, we investigate the tensor charges in the SU(3) chiral quark-soliton model which is also called the semibosonized SU(3) Nambu-Jona-Lasinio model. For the first time, the octet tensor charge $g_{\mathrm{T}}^{(8)}$ and hence the net number of the transversely polarized strange quarks in a transversely polarized nucleon δs are calculated. An interesting feature of our model is that it predicts the negative nonzero number of the polarized strange quarks Δs in the longitudinally polarized nucleon [30,31], which is consistent with the corresponding experimental value, whereas it yields the number of the transversely polarized strange quarks δs in a transversely polarized nucleon compatible with zero.

The dynamical origin of the difference between the axial and tensor charges in our model can be related to the qualitatively different behaviour of the charges with soliton size [15]. The detailed discussion of this issue will be published elsewhere.

TABLE I. Tensor charges $g_{\mathrm{T}}^{(0)}$, $g_{\mathrm{T}}^{(3)}$ and $g_{\mathrm{T}}^{(8)}$ with the constituent quark mass $M = 420$ MeV. The current quark mass m_s is chosen as $m_s = 180$ MeV. The final model predictions are given by $\mathcal{O}(\Omega^1, m_s^1)$.

	$\mathcal{O}(\Omega^0, m_s^0)$	$\mathcal{O}(\Omega^1, m_s^0)$	$\mathcal{O}(\Omega^1, m_s^1)$
$g_{\mathrm{T}}^{(0)}$	0	0.69	0.70
$g_{\mathrm{T}}^{(3)}$	0.79	1.48	1.54
$g_{\mathrm{T}}^{(8)}$	0.09	0.48	0.42

TABLE II. Each flavour contribution to the tensor charges as varying the constituent quark mass M. The current quark mass m_s is chosen as $m_s = 180$ MeV.

M [MeV]	δu	δd	δs
370	1.18	-0.41	0.002
400	1.14	-0.41	-0.004
420	1.12	-0.42	-0.008
450	1.12	-0.41	-0.02

REFERENCES

[1] For a review and references, see J. Kodaira, in : Proceedings if the YITP workshop, Kyoto, Japan, October 1994.

[2] J. Ralston and D. Soper, Nucl. Phys. **B152**, 109 (1979).

[3] R.L. Jaffe and X. Ji, *Phys. Rev. Lett.* **67** (1991) 552; *Nucl. Phys.* B375,527 (1992).

[4] X. Ji, *Phys. Lett.* **284B**, 137 (1992).

[5] R.L. Jaffe and X. Ji, *Phys. Rev. Lett.* **71**, 2547 (1993).

[6] C. Bourrely and J. Soffer, *Nucl. Phys.* **B243**, 329 (1994); *Nucl. Phys.* **B445**, 341 (1995).

[7] RSC collaboration, G. Bunce *et al.*, Particle World **3** (1992) 1; Proposal on Spin Physics using RHIC Polarized Collider, R5 (August 1992), approved October 1993.

[8] The HERMES proposal to HERA (1993).

[9] COMPASS Collaboration (G. Baum et al.). CERN-SPSLC-96-14 (1996).

[10] R.L. Jaffe and X. Ji, Phys. Rev. Lett. **67**, 552 (1991).

[11] H. He and X. Ji, Phys. Rev. **D52**, 2960 (1995).

[12] J. Kodaira, S. Matsuda, K. Sasaki and T. Uematsu, *Nucl. Phys.* **B159**, 99 (1979).

[13] A.P. Bukhvostov, E.A. Kuraev and L.N. Lipatov, *Sov. Phys. JETP* **60**, 22 (1983).

[14] X. Artru and A. Mekhfi, Z. Phys. **C45**, 669 (1990).

[15] H.-C. Kim, M.V. Polyakov, and K. Goeke, Phys. Rev. **D53**, R4715 (1996).

[16] D.Diakonov and M.Eides, *Sov. Phys. JETP Lett.* **38** (1983) 433

[17] I.J.R. Aitchison, C.M. Fraser, *Phys. Lett.* **146B**, 63(1984).

[18] A.Dhar, R.Shankar and S.Wadia, *Phys. Rev.* **D31** (1985) 3256

[19] D. Dyakonov and V. Petrov, *Nucl.Phys.* **B272**, 457 (1986); preprint LNPI-1153), published in: *Hadron matter under extreme conditions*, p.192, Kiew (1986).

[20] D. Diakonov, V. Petrov, and P.V. Pobylitsa, Nucl. Phys. **B306**, 809 (1988).

[21] S. Kahana, G. Ripka and V. Soni, *Nucl.Phys.* **A415**, 351 (1984).

[22] S. Kahana and G. Ripka, *Nucl.Phys.* **A429**, 962 (1984).

[23] M.S. Birse and M.K. Banerjee, Phys. Lett. **136B**, 284 (1984).

[24] E. Witten, *Nucl.Phys.* **B223**, 433 (1983).

[25] C.V. Christov, A. Blotz, H.-C. Kim,P. Pobylitsa, T. Watabe, Th. Meissner, E. Ruiz Arriola and K. Goeke, Prog. Part. Nucl. Phys. **37** to be published (1996).

[26] M. Wakamatsu and T. Watabe, *Phys. Lett.* **312B**, 184 (1993).

[27] C.V. Christov, A. Blotz, K. Goeke, P. Pobylitsa, V. Petrov, M. Wakamatsu, and T. Watabe, *Phys. Lett.* **325B**, 467 (1994).

[28] M. Praszałowicz, A. Blotz and K. Goeke, Phys. Lett. **354B**, 415 (1995).

[29] A. Blotz, D. Diakonov, K. Goeke, N.W. Park, V. Petrov and P.V. Pobylitsa, Nucl. Phys. **A555**, 765 (1993).

[30] A. Blotz, M. Praszałowicz, and K. Goeke, *Phys. Rev.* **D53** (1996) 485.

[31] A. Blotz, M. Polyakov and K. Goeke, *Phys. Lett.* **302B**, 151 (1993).